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I conceive that I have now established the positions with which I set out. I have *proved* that the expression usually assigned as the probability of one life surviving another is *rigorously true* on the hypothesis of a uniform distribution of the deaths of each year; and I have shown, by comparison of certain results of that hypothesis with the corresponding results of another, and unquestionably a more correct hypothesis, that the error arising from the employment of the first-mentioned hypothesis is so small as seldom to require to be taken into account. It will therefore now be for such of those who have followed me thus far, as demur to my conclusions, to show the fallacy of my demonstrations and the inconclusive character of my reasonings. Till this be done, I submit that I am warranted in viewing the questions in debate as set at rest.

London, 18th November 1850.

On the Contrivances required to render contingent Reversionary Interests Marketable Securities.

[Continued from No. 2.]

SINCE the loss occasioned by the reassurance varies with the mode of operation, it becomes interesting to inquire in which way the loss may be made the least possible. B is in the situation of a merchant who possesses a commodity not quite in a marketable state, and who seeks to bring it into a condition for sale at the least expense. A little reflection will show that the longer the borrowing is deferred, the less loss will B sustain, on account of the difference at interest; hence we must seek to diminish as much as possible the early payments, and to throw the weight of the premiums to a distant time—a scale of ascending premiums is thus the more advantageous for B. Now the most rapid rate of ascent which an assurance office can allow, is such that each premium may pay for the succeeding year's risk; no office, indeed, will go thus far*; yet it is worth while to examine this case, since it exhibits the minimum loss at which B can procure a reassurance.

CASE 4.—Let the assurance of £1, payable at the death of B,

* Some will.—ED. A. M.

if it happen while A is alive, be purchased from year to year at the price of each year's risk.

The successive payments, computed at 3 per cent., are given in the subjoined table:—

Ages.	Premium.	Ages.	Premium.
30 58	·00983	54 82	·01548
31 59	·00991	55 83	·01633
32 60	·00981	56 84	·01723
33 61	·00973	57 85	·01879
34 62	·00982	58 86	·02154
35 63	·00991	59 87	·02485
36 64	·01019	60 88	·02937
37 65	·01048	61 89	·03146
38 66	·01077	62 90	·03206
39 67	·01144	63 91	·03230
40 68	·01252	64 92	·03370
41 69	·01324	65 93	·03524
42 70	·01380	66 94	·03664
43 71	·01395	67 95	·03863
44 72	·01408	68 96	·04079
45 73	·01402	69 97	·04301
46 74	·01394	70 98	·04544
47 75	·01370	71 99	·05271
48 76	·01302	72 100	·05967
49 77	·01276	73 101	·06598
50 78	·01250	74 102	·07108
51 79	·01325	75 103	·06275
52 80	·01407	76 104	·05073
53 81	·01485		

And computing the present values of these contingent payments at 6 per cent., their sum comes to ·10022, which deducted from ·50280, leaves for B the balance ·40258. The loss occasioned by this form of the transaction is only ·00143.

This loss is scarcely one-third part of that occasioned by uniform annual premiums; the amount of it might have been readily obtained from other considerations, which throw additional light upon the influence exerted by the one transaction on the other.

The premium for one year's assurance depends very little on the rate of interest, and is just the probability of the death occurring within the year discounted for half a year; wherefore the numbers in the preceding table multiplied by $\sqrt{1{\cdot}03}$ would

give the probabilities of B dying before A in each successive year. These probabilities, divided by $\sqrt{1{\cdot}06}$, would be the premiums exacted if the office did business at 6 per cent. Hence the 3 per cent. premiums multiplied by $\sqrt{\frac{1{\cdot}03}{1{\cdot}06}}$ would give the 6 per cent. premiums.

Now the 6 per cent. premium, estimated at 6 per cent., would just give for present value that of assurance at B's death if first, that is ·09879. Hence the present value of the 3 per cent. premiums must be $\cdot09879 \times \sqrt{\frac{1{\cdot}06}{1{\cdot}03}}$, so that the loss to B occasioned by this form of the transaction is $\cdot09879 \left\{\sqrt{\frac{1{\cdot}06}{1{\cdot}03}} - 1\right\}$ $= \cdot00143$; that is, just about $1\frac{1}{2}$ per cent. on ·09879. It is as if the amount ·09879 were operated on for half a year at the two rates of interest. Hence if the premium were paid half-yearly or quarterly, according to the same plan the loss would be still further reduced; until, if we imagine the price to be paid each morning for the day's risk, the loss would be reduced to zero.

These discrepancies among the results of the various methods of effecting reassurances lead one to consider whether there be any intrinsic quality in contingent reversions which may render their conversion into absolute reversions a matter of necessity; and whether any distinct principle can be discovered which may serve to regulate the mode of conversion, or the difference between the market rates of interest to be charged for advances on the two kinds of security. In a strictly logical view, we cannot draw any line of demarcation between the two, for in either case the purchaser incurs the risk of losing a sum of money, whether it be the interest alone or the capital and interest jointly, and in return for this risk enjoys the chance of obtaining more than the sum expended with interest thereon; neither indeed is there any generic distinction recognized in practice, since even the scheme of reassurance shows that by the addition of two contingent reversions an absolute reversion is produced.

That there is no intrinsic quality in contingent reversions to render a higher rate of interest indispensable, may be made

evident by the following view of two complementary transactions:

A property has been bequeathed in life-rent to A, and afterwards in reversion to B if B be alive at A's death, but if not to fall to C as absolute residuary legatee.

If, now, the two parties B and C come to an arrangement, they will be able jointly to offer for sale an absolute reversion to fall due at the death of A. For this the office would give the value computed at 6 per cent. without reassurance. But if B come alone and offer for sale his contingent reversion, then a higher per-centage on the expense of a reassurance is charged, while if C also separately sell his contingency, a similar charge is made: that is, on the supposition that the two kinds of reversions ought to be computed differently, the sum of the values of the two expectations is not equal to the value of the sum of those expectations.

Although, however, it appear from the above reasoning that there is no intrinsic distinction between absolute and contingent reversions as valued under fair and average circumstances, we must not conclude that there is nothing in the circumstances of the case which may call for a change in the manner of computation. It is well known that the advantage of selection is considerable, especially where the parties have an intimate knowledge of the habits and constitution of the nominee, and that that advantage is greater when there are two lives involved than when there is only one. The certificates of health only partially meet this advantage, and it is almost established that whenever a private bias operates extensively, all the care of the offices is insufficient to counteract it. In the above case we may imagine that A is a hale, temperate old man, whose years may be expected to run considerably beyond the average for his age. The two heirs B and C therefore agree together and sell their joint expectation; against this selection the office has no defence except by charging a higher rate. But if B and C should not agree, B may, feeling himself to be not better than an average life, sell his separate expectation of succeeding to A. Here there is the advantage of two selections, and at the same time one of these greatly enhanced in value by the mere introduction of a second life. Supposing the chance of unhealthiness in B to be met by the medical examination, still the benefit of selection would be greater in the contingent than in the abso-

lute reversion, and therefore, viewing the subject in its whole extent, the offices are reasonably entitled to charge different rates for the two classes of reversions.

If the values of selection at different ages were known, the fair and proper charges could thence be computed without any change in the rate of interest, and the results so brought out would be intelligible and consistent with each other. The incongruity among the results arises from our being compelled to meet the selection in an indirect and not altogether proper way.

It is a matter of regret that the mutual jealousies of the offices prevent the accumulation of their experience on these points, and thus tend to cramp considerably the management of their business. The influence of selection cannot be represented by any bill of mortality. Thus the selection of A in the above case would not be properly allowed for by taking the mortality for the class of annuitants; but the calculation is of a special nature, having as its most important element the age of the party at the date of selection. On this important branch of life-statistics there are hardly any available data, and the reporter is therefore unable to give any satisfactory information.

From the above investigations, it will be seen that there are in general two methods of making the computations; one of these is to calculate the value of the absolute reversion formed, by adding together the original contingency and the assurance, and from that value to deduct the present value of the premiums to be paid to the assurance office, discounting these at 6 per cent. and allowing for the expectation of human life.

The second method is to take the difference between the premium actually charged by the office and that which would result from calculations at 6 per cent. The present value of that difference, estimated as above, is the loss arising from reassurance, which loss falls to be deducted from the value of the expectation actually possessed by B.

Having thus obtained the present advance to be made in consideration of a contingent reversion of £1, we must divide the proposed present advance thereby, in order to discover what sum must be secured.

In the above cases we have the following results:—

1. To cover an advance of £100, and also of the purchase-money (computed at 3 per cent.) for an assurance at the death of A, provided B be then dead, and also to cover interest at

6 per cent. for both advances, the sum of £273·93 must be secured at A's death if first, and also by assurance at A's death if second.

2. To cover a present advance of £100, and also of the price of an assurance at the death of B if it happens before A's, the sum of £266·31 must be secured, the assurance being also taken for that amount.

3. For the present advance of £100, and the current advance for the annual premium for assurance at the death of B if first, and to provide interest at 6 per cent. for those advances, there must be secured at the succession of B to A, and also by assurance at the death of B if first, the sum of £250·39.

4. For the same advance of £100, and of premiums according to the mode of successive yearly assurance, with interest at 6 per cent., there must be secured £248·40.

5. To cover an advance of £100 with interest at 6 per cent. without reassurance, there must be secured at B's succession to A the sum of £247·52.

The second question proposed to Mr. Sang is as follows:—

B, aged 30, wishes to borrow £100, to be repaid on his succession to A, aged 58, to whom he is heir of entail. In this case the office deem it proper to reassure on B's death, whether that event may happen before or after A's, in order that they may secure themselves out of the rents, should B be unable to pay the money down. Required the amount to be secured, and also an opinion as to the vesting of the policy of assurance should B pay up the stipulated sum.

Opinion.

In this question it is assumed that B being only heir of entail, and therefore life-renter of the property, may at the time of his succession be unable to pay the stipulated sum. Besides the reassurance at the death of B, if it should happen before A's, the lender requires an assurance at B's death, should it happen after he has succeeded to the property. The money may thereafter be regarded as lent to B by way of redeemable annuity, he paying the interest of the money and the premium for assurance.

Before any computation can be gone into, it must first be clearly understood what party is entitled to the policy when the money has been repaid. Up to the date of B's succession to

the property, the policy is clearly for the benefit of the lender. To bring out the points of the case, let us suppose that B succeeds to the property, pays the whole sum, and shortly after dies. The lender, if the policy belong to him, would then receive the money twice over, and would seem to be overpaid. The injustice is however only apparent, its reality depending entirely on the nature of the original agreement. Nay, it may happen, should the succession be long of opening, that the lender, having paid a great number of premiums, loses by the reassurance. The best way of examining the subject is to consider separately two cases; the one, in which it is agreed that the policy of assurance is to remain the property of the lender; the other, in which the policy is to be assigned to B whenever he repays the money.

Case 1.—B, heir of entail to A, wishes to borrow £100, to be repaid at his succession to the property, the policy of assurance however remaining the property of the lender after redemption. Required what sum must be secured.

For convenience, let us first compute what present advance must be made for £1 so secured.

In this case the lender agrees to purchase the reversion of £1 at the succession of B to A, but from the imperfect nature of the security, only on condition that B procure and assign a policy of assurance at his death, the lender making his valuations at 6 per cent.

The 3 per cent. premium for this assurance is ·01981; and the lender having to disburse this as it falls due, estimates its present value at ·27773, obtained by multiplying the actual premium by the 6 per cent. value of an annuity during the life of B. In return for this the lender has the expectation of £1, to be paid at the death of B, which expectation, valued at 6 per cent., is worth ·21252. According to the lender's calculation, therefore, he has a loss on this transaction of ·06521.

The 6 per cent. value of £1 due at the succession of B to A is ·40401, which value B is to receive, subject however to the deduction of the above loss. The balance is thus ·33880.

Hence for £100 of present advance there must be secured at B's succession, and also at B's death, the sum of £295·16, the lender paying the annual premium £5·847 during the life of B, and drawing the sum assured at his death.

If at the death of A, B discharge his obligation, the lender

may dispose of the policy, or may keep it up, as he may be inclined. Should B not discharge the debt, but offer to pay in lieu a redeemable annuity, the existing policy may be made available, but still on the distinct ground that it is the property of the lender. The manner of effecting this is of course open to arrangement between the parties. Two courses present themselves:—

1st. B may pay annually the interest of the £295·16, together with the premium for assurance, at his then age, to the lender, out of which premium the smaller premium charged by the office may be taken, leaving the difference of the premiums as payment to the lender for the value of the policy: by this arrangement the lender would claim the proceeds of the policy at the death of B; while if B should redeem the original bond, he would need also to compound for this difference of premiums, and so obtain the policy for himself.

2ndly. B may at his succession purchase the policy at its then value, continue the payment of the premium with interest by way of redeemable annuity, assigning the policy to the lender in security.

3rdly. B may augment his debt by the value of the policy, and pay interest for the debt so augmented, together with the premium; but this arrangement would require the opening of a new policy for the augmentation of the debt, and could only be gone into if the borrower were in good health at the time.

Case 2.—B, heir of entail to A, wishes to borrow £100, to be repaid at his succession to the property, to be additionally secured by a policy of assurance, which policy is to become the property of B after redemption; and in case that the obligation be not redeemed at the date of the succession, the borrower is to continue the payment of the premium with interest until redemption, the policy remaining with the lender in security until then.

In this case the lender is secured in the repayment of the principal sum at the occurrence of the first death of A or B; the arrangement for a redeemable annuity after the succession being equivalent to payment of the principal sum on that event. But the policy of assurance conveys more than that expectation; the value of the additional security thereby afforded falling to be handed over to B at A's death, and the borrower being thus enabled, out of the rents of the property, to secure his personal

representatives from the repayment of the advance. The terms must, of course, be proportionably higher.

The expectation of £1 at the first death of A or B, computed at 6 per cent., is worth ·50280, while the present value of the premium, payable during the joint life, is ·17906, leaving to B a balance of ·32374.

At this rate, for a present advance of £100, there will need to be secured the principal sum of £308·89: but then the lender is bound to pay an annual premium of £6·119 during B's life until his succession to A, and then to hand over to B the policy thus expensively kept up, which policy may, if the succession open late, be of very considerable value.

The difference between the results brought out in the last two cases, shows the propriety of having the destiny of the policy of reassurance clearly laid down in the contract, and of having the computations made according to the very conditions therein contained.

(To be concluded in our next Number.)

On the Determination and Division of Surplus, and on the Modes of returning it to the Contributors.

[Concluded from No. 2.]

HAVING, in our preceding Number, endeavoured to lay down what appears to us to be a correct method of procedure with reference to the determination and division of surplus, we now proceed to consider the modes usually adopted for the return of it to the contributors. Of these we believe the following to be the principal ones practised at the present day:—

1. The share of surplus is returned by way of reduction in the premium next payable.
2. By way of reduction in the t next premiums payable.
3. By way of reduction in all the future premiums.
4. In augmentation of the sum assured.
5. In augmentation of the sum assured, but at a certain rate per cent. per annum thereon.
6. In any one of these ways, but subject to the assured having